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Book of Abstracts

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<i>S. Stepaniuk</i>	
Secret sharing schemes	76
<i>K. Ya. Sysak, M. S. Sidorov, I. S. Klimenko</i>	
Metabelian Lie algebras of derivations of rank two	77
<i>A. V. Tarasevych</i>	
On generalized ternary quasigroup functional equations of the type $(5; 3; 0; 0)$	78
<i>R. F. Tsimbolinec, A. A. Tylyshchak</i>	
Irreducibility of some 8×8 -matrices over local ring of finite length	79
<i>Lyudmila Turowska</i>	
Weighted Fourier algebras and complexification	80
<i>A. V. Tushev</i>	
On primitive modules over group algebras of linear groups	81
<i>V. O. Ustimenko</i>	
On families of stable subsemigroups and subgroups of affine Cremona semigroups with exponential growth of periods and non-commutative cryptography	82
<i>V. O. Ustimenko, O. S. Pustovit</i>	
On noncommutative cryptography responds to challenges of quantum computer and secure big data processing	83
<i>Yaroslav Vorobets</i>	
Topological full group containing the Grigorchuk group	84
<i>B. V. Zabavsky, O. V. Domsha, O. M. Romaniv</i>	
Clear elements and clear rings	85
<i>B. V. Zabavsky, O. B. Popadiuk</i>	
Simple element of a Bezout domain	86
<i>Efim Zelmanov</i>	
Finitely presented algebras	87
<i>A. V. Zhuchok</i>	
The least dimonoid congruence on the free g -dimonoid	88

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Clear elements and clear rings

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Throughout the paper we suppose R is an associative ring with non-zero unit and $U(R)$ its group of units. The vector space of matrices over the ring R of size $k \times l$ is denoted by $R^{k \times l}$ and group of units of the ring $R^{n \times n}$ by $GL_n(R)$. A ring R is called an *elementary divisor ring* [1] if for an arbitrary matrix A of order $n \times m$ over R there exist invertible matrices $P \in GL_n(R)$ and $Q \in GL_m(R)$ such that PAQ is a diagonal matrix (d_{ij}) for which d_{ii} is a total divisor $d_{i+1,i+1}$ (i.e. $Rd_{i+1,i+1}R \subseteq d_{ii}R \cap Rd_{ii}$) for each i . An element a of ring R is *clear* if $a = r + u$ where r is unit-regular element and $u \in U(R)$. The ring R is *clear* if every its element is such.

Theorem 1. *Let R be a commutative elementary divisor ring and A is a full nonsingular matrix of $R^{2 \times 2}$. Then exist invertible matrices $P, Q \in GL_2(R)$ such that PAQ is nontrivial clear element of $R^{2 \times 2}$.*

Theorem 2. *Let R be a commutative elementary divisor ring. Then every full nonsingular matrix $A \in R^{2 \times 2}$ is nontrivial clear.*

Theorem 3. *Let R be a semi-simple commutative Bézout domain. The next statements are equivalent:*

1. *R is an elementary divisor ring;*
2. *any full nonsingular matrix of $R^{2 \times 2}$ is nontrivial clear.*

Theorem 4. *Let R be a commutative Bezout domain. Ring $R^{2 \times 2}$ is a clear if and only if $R^{2 \times 2}$ is 2-good ring.*

Some open questions.

1. Is the commutative clear ring a ring of unit-regular stable range 1?
2. Is the notion of a ring of unit-regular stable range 1 a left-right symmetric?

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